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Relativistic causality and superluminal signalling using X-shaped localized waves

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Abstract. It has been established theoretically and experimentally that X-shaped localized waves have peaks that travel at superluminal speeds. A study of the excitation of such pulses has shown that their peaks undergo a delayed generation before they are launched. Consequently, these peaks travel at superluminal speeds for a finite distance beyond which they propagate at the speed of light. We demonstrate that this local superluminal propagation does not constitute a violation of the theory of special relativity in a global sense. The use of this local superluminality in signalling is investigated and implications pertaining to causality are discussed.

1. Introduction

In recent experimental and theoretical investigations, it has been shown that X-shaped localized waves (or pulsed Bessel beams) have peaks travelling at superluminal speeds [1–8]. An example of such wave solutions is the focused X-wave that has been deduced using the superluminal boost representation [6]. Explicitly, the focused X-wave solution has the form

$$\Psi_{\mathrm{FXW}}(\rho, z, t) = \frac{1}{\sqrt{\rho^2 + (a_1 + \mathrm{i}\gamma(z - vt))^2}} \mathrm{e}^{-\kappa_0\sqrt{\rho^2+(a_1+\mathrm{i}\gamma(z-vt))^2}} \mathrm{e}^{+\mathrm{i}\kappa_0\gamma((v/c)z-ct)} \tag{1}$$

where $\gamma = 1/\sqrt{(v/c)^2 - 1} > 0$ and $(v/c) > 1$. Here, κ_0 and a_1 are constants that determine the shape of the pulse and its frequency bandwidth. In the limit $\kappa_0 \to 0$, the FXW reduces to the X-wave solution introduced by Lu and Greenleaf [1]. The zeroth-order X-wave has the following explicit form:

$$\Psi_{\mathrm{XW}}(\rho, z, t) = \frac{1}{\sqrt{\rho^2 + (a_1 + \mathrm{i}\gamma(z - vt))^2}}. \tag{2}$$

The wave solutions given in equations (1) and (2) have peaks that move with superluminal velocities. This property has invoked claims to the possible breakdown of the theory of relativity [9–11]. Before one can judge such serious allegations, a better understanding of the propagation and generation of these superluminal wave solutions is necessary. A careful

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study of various configurations that can produce X-shaped pulses similar to those given in equations (1) and (2) has been provided in [12]. It has been shown that X-shaped localized waves have peaks that acquire superluminal speeds after a generation delay at the aperture plane. The subsequent superluminal propagation is observed over a finite range beyond which the speed of the pulse asymptotically approaches the speed of light. Consequently, an X-shaped pulse acquires local superluminal speeds, while its peak always falls behind an ordinary wave travelling at the speed of light and undergoing no delayed generation. As such, X-shaped localized waves cannot be used in superluminal signalling in a global sense. Instead, they acquire superluminal speeds locally over a finite range. We believe that it is timely to study the effect of this local superluminality on relativistic causality, especially after the generation of superluminal X-waves and Bessel X-pulses using optical [4, 13] and microwave sources [8].

The plan of this work is to show in section 2 that the superluminal speed of the peak of an X-shaped pulse is local in character and does not allow the global superluminal transmission of signals. We also discuss a scheme that may be used for local superluminal signalling. In section 3, we summarize Bohm's relativistic argument against superluminal signalling [14]. We then specify the constraints that prevent signalling to the past from happening. Our concluding remarks are provided in section 4.

2. Local versus global superluminality

The results of [12] indicate that local transmission of superluminal X-shaped pulses is possible. Such a prospect leads to speculations concerning using X-shaped pulses for superluminal signalling. This is a crucial issue, especially when considering ultra-short pulses exhibiting few oscillations. In such a case, the propagating pulse is very narrow and the superluminal propagation of the peak of the pulse is manifest (cf figures 7 and 11 in [12]). We have already demonstrated that X-shaped pulses generated using different schemes exhibit a 'delayed generation' followed by a superluminal 'catching up' [12]. One can, thus, represent the propagation of ultra-short X-shaped pulses by the Minkowski diagrams shown in figure 1. This figure shows three possible courses of action of the generated X-shaped pulses. These three schemes do not violate the special theory of relativity. The diagram displayed in figure 1(*a*) depicts a pulse generated after a time delay T_0; henceforth, it propagates at a superluminal velocity that asymptotically approaches c. At the finite distance L, the deviation of the velocity of the pulse from c is negligible. The distance L can thus be defined as the distance at which the deviation of the speed of the pulse from c is not measurable. The diagram, shown in figure 1(*a*), is a schematic representation of the propagation of an X-shaped pulse generated

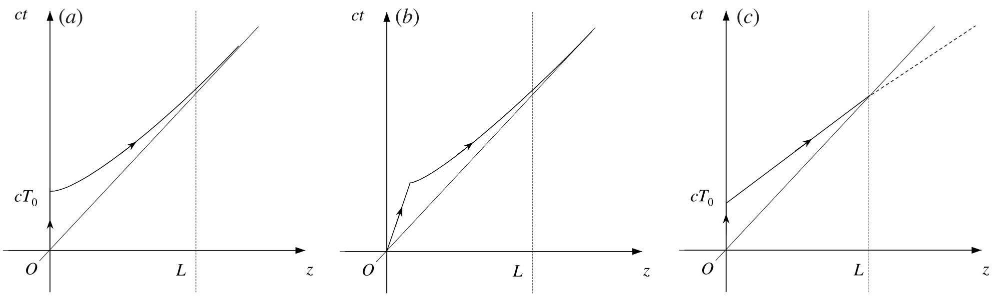

Figure 1. Minkowski diagrams representing various situations depicting 'delayed generation' followed by superluminal propagation.

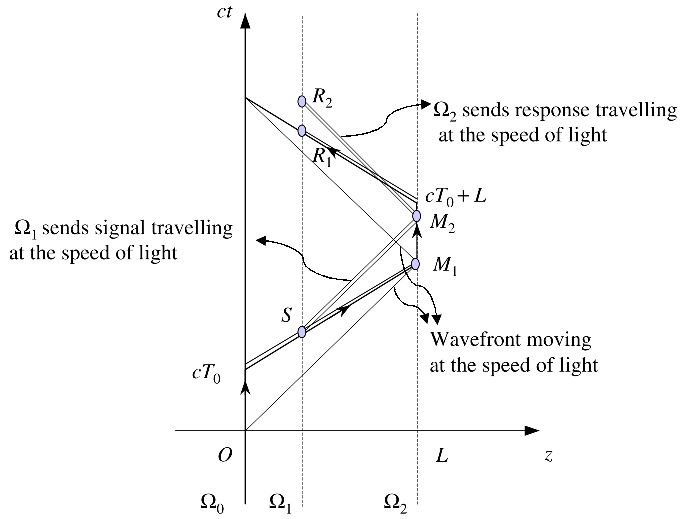

Figure 2. Proposed scheme for superluminal signalling using a shutter operator. Observer Ω_1 uses shutter (S) to transmit information using a locally superluminal pulse train. Observer Ω_2 receives the signal at M_1 and responds with superluminal signal that reaches Ω_1 at R_1. The signals transmitted between Ω_1 and Ω_2 are received at R_2 and M_2, respectively.

using an annular slit or a refractive axicon [12]. For the first source, the delay is due to the time taken by the fields generated at the annulus to reach its axis where they interfere, hence, forming the X-shaped pulse. On the other hand, the delay associated with a refractive axicon is due to the time consumed by the pulse as it goes through the material of the axicon. The diagram, shown in figure 1(b), represents the propagation of a pulse generated by a diffractive axicon. The initial propagation of the pulse is subluminal. It then becomes superluminal for the remaining part of the distance L. Finally, figure 1(c) displays an abstract situation where a delayed generation of the pulse is followed by propagation at a constant superluminal velocity. Such propagation is upheld for a finite range L beyond which the pulse dissipates very quickly or the pulse becomes indistinguishable from the surrounding background field. This abstract representation holds a close resemblance to the more physical ones displayed in figures 1(a) and (b), and is easier to handle graphically when issues pertaining to the question of causality are addressed. Furthermore, this abstract representation can serve as a general model for any scenario (not necessarily that of the X-shaped pulses considered in [12]) in which local superluminality is achieved due to 'delayed generation' followed by some 'catching up' over a finite distance.

In all three situations illustrated in figure 1, the global transmission of the pulses occurs at velocities smaller than c. Therefore, the propagation of these pulses is consistent with the general framework of the theory of special relativity. Locally, however, one can observe superluminal transmission of ultra-short pulses. Evidently, one should wonder whether such local transmissions can be used for transmitting superluminal signals. A contraption that could make use of this local superluminal transmission is a gate or a shutter placed at an intermediate position between the source and a receiver. Assume that the source Ω_0 is transmitting a wavetrain of pulses separated at known time intervals. Individual pulses within such a wavetrain are stopped/passed at will by an operator of the shutter Ω_1, as shown schematically in figure 2. The action of the operator (signal) could be thus transmitted at superluminal speed to the receiver Ω_2. There are several points that have to be checked before such apparently simple system is deemed feasible. An important factor that can affect the performance of the suggested system is that the movement of the shutter and its positioning in front of the source may disturb the propagation of the transmitted pulses. This can be especially critical since the superluminal propagation is more prominent in the near-field range. If the shutter were placed further away from the source, the observable deviation from the speed of light would be minimal. On the other hand, placing the shutter too close to the source could affect the delicate coherence of the wave fields radiated from different sections of the source plane. The interference of these wave fields gives rise to the superluminal X-shaped pulses. The shutter might, thus, affect their coherence to the extent of destroying the pulses altogether, or dissolving their superluminal character. It must be remembered, however, that any argument disclaiming the possibility of using a shutter for transmitting superluminal signals is speculative until an accurate simulation of the action of the shutter is undertaken. Alternatively, an experimental test of this procedure could be a worthwhile endeavour.

The preceding discussion implies that the use of the local superluminal transmission of the X-shaped pulses depends on the possibility of having a shutter device that would not destroy the transmitted pulses or their superluminal character. Until such a crucial issue is resolved, the question of causality remains unsettled. In particular, can one signal to the past if the aforementioned scheme were feasible? Can we reach extreme situations of having 'delayed generation' at $cT_0 = L$, such that the subsequent local transmission of the pulses is instantaneous? Will the need for preserving causality set any limits on the velocity of the local transmission of pulses? Before discussing any possible violation of causality, one should recall that the theory of relativity is based upon two principles, namely, the constancy of the speed of light and the invariance of the laws of physics in all inertial frames. Superluminal wave propagation has been dismissed, within the framework of special relativity, because we can use it to signal to our own past. Therefore, superluminal signalling violates causality, while it does not *per se* contradict the assertions of special relativity, or, as claimed, 'implies the breakdown of the principle of relativity'. In fact, there are attempts to generalize the theory of special relativity in a consistent way by allowing for both superluminal and subluminal speeds without violating causality [15–20]. Incidentally, in [15], it has been predicted that the simplest superluminal object is a rigidly moving X-shaped wave. Within the framework of 'extended relativity', it has been argued that single tachyon superluminal transmission does not violate causality [18–20]. The preservation of causality in such an approach depends on the Feynman–Stückerlberg 'switching procedure', which involves the switching of the observation of tachyons and antitachyons depending on the frame of reference of the observer. For an optical signal, 'extended relativity' anticipates that antiparticles of polarized photons are photons having opposite helicities. In this work, we choose to limit our analysis to the framework of ordinary special relativity. We demonstrate that optical X-shaped pulses having peaks that travel at superluminal speeds for a finite distance do not violate special relativity. Furthermore, we show that the use of such pulses for superluminal signalling allows us to communicate with our past. However, this can only happen under very stringent conditions.

3. Local superluminality and relativistic causality

Ambiguity in the order of time arises in special relativity when two events are outside each other's lightcones. Essentially, different observers will not agree on which one of the two events occurs earlier [14]. This causes severe difficulties with causality, as one observer can note that one event is the cause of the second, while another observer can reverse their order. This ambiguity is usually removed if no influence can be transmitted at a velocity larger than c. In the case where such a condition is not imposed, it becomes possible to signal to the

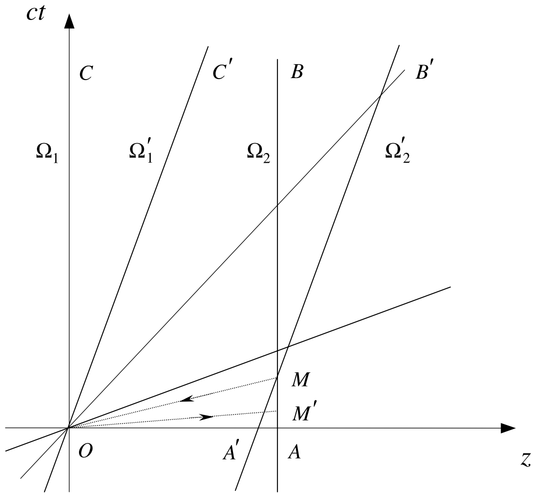

Figure 3. Minkowski diagram of Bohm's construction illustrating possible transfer of information to the past if superluminal signalling is available.

past. This point has been candidly discussed in [14]. Bohm's argument, illustrated in figure 3, is based on the possibility that observers Ω_1 and Ω_2 would come into physical contact with observers Ω_1' and Ω_2', respectively. It is assumed that during such a contact information can be transferred between the observers without delay. Relative to the frame of reference of Ω_1 and Ω_2, the other two observers Ω_1' and Ω_2' are moving at a constant speed $v < c$. The observer Ω_2' (represented by the worldline $A'B'$) gets in contact with Ω_2 (represented by the worldline AB) at the point M. During this contact, a piece of information is passed from Ω_2 to Ω_2', which is subsequently transmitted at a superluminal speed to observer Ω_1'. If Ω_1' receives this information at point O, or at any time prior to reaching this point, he can transfer this information to observer Ω_1. The latter can send back a superluminal signal to Ω_2 reaching him at M'. Evidently, $ct(M') < ct(M)$ and the described scheme enables observer Ω_2 to send specific information to his own past. As such, any superluminal signal should be ruled out in order to preserve causality.

It is of interest to examine the above argument against the possibility of having signals undergoing local superluminal transmissions (cf figures 1 and 2). As shown in figure 4, observer Ω_1 is assumed to be able to transmit a local superluminal signal that has a 'delayed generation' time $cT_0 = \alpha L$ and a finite signal duration $c\tau$. An identical transmitter is also used by observer Ω_2' who is moving at relative speed $v < c$. Both Ω_2' and Ω_1 are shutter operators that are assumed to be very close to the generators of trains of pulses of the type illustrated in figures 1 and 2. They are chosen to be as close as possible to the generators because for such positions the likelihood of violating causality is maximized. Now following Bohm's argument, we assume that upon contact between Ω_2 and Ω_2' a piece of information is passed from the former to the latter. Such information is immediately transmitted by Ω_2' to Ω_1'. This signal should be received entirely by Ω_1' before he gets in contact with Ω_1. When the two get in contact with each other, the piece of information is passed from Ω_1' to Ω_1. The latter transmits the information back to Ω_2. From figure 4, one may conclude that in order to preserve causality, the transmission from Ω_1 to Ω_2 must end before Ω_2' gets in contact with Ω_2 (after that contact Ω_2' starts his transmission to Ω_1'). This condition implies that $ct(E_1) < ct(E_2)$. Alternatively, one can conjecture that causality necessitates that the slope (S') of the signal from Ω_2' to Ω_1' should be smaller than the slope (S) of the signal from Ω_2 to Ω_1. The two conditions $ct(E_1) < ct(E_2)$ and $S' < S$ are not equivalent; however, they lead to qualitatively similar results. One should note that although the first condition relates to realistic situations,

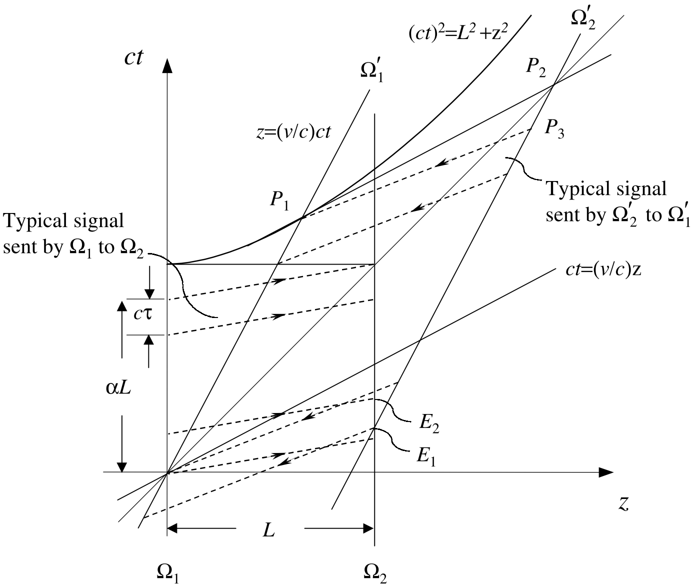

Figure 4. Bohm's construction examined for the case of local superluminal signalling by pulses undergoing 'delayed generation' followed by superluminal 'catching up'.

the second one is more severe. We choose to examine the second condition in order to illustrate certain points of interest. Toward such a goal, we need to evaluate the slopes S and S'. From figure 4, it is straightforward to show that

$$S = (1 - \alpha). \tag{3}$$

To determine S', we calculate the coordinates of the points P_1, P_2 and P_3. This is done by evaluating the intersection between $z = (v/c)ct$ and $(ct)^2 = L^2 + z^2$, which yields $P_1 = (\beta\gamma L, \gamma L)$. Here, $\beta = (v/c)$ and $\gamma = 1/\sqrt{1-\beta^2}$. The point P_2 connects to P_1 through a straight line of slope β; accordingly, one can show that $ct_2 = z_2 = L\sqrt{(1+\beta)/(1-\beta)}$. The length $\overline{P_1P_3}$ represents the period $(1-\alpha)L$ transformed to the primed frame. After some manipulation, we obtain $z_3 = L\sqrt{(1+\beta)/1-\beta} - L\beta\gamma(1-\alpha)$ and $ct_3 = L\sqrt{(1+\beta)/1-\beta} - L\gamma(1-\alpha)$. Thus, the slope of $\overline{P_1P_3}$ is given by

$$S' = \frac{\beta - (1-\alpha)}{1 - \beta(1-\alpha)}. \tag{4}$$

From equations (3) and (4), one can conclude that causality necessitates that

$$(1-\alpha) > \frac{\beta - (1-\alpha)}{1 - \beta(1-\alpha)} \tag{5}$$

or

$$f(\alpha, \beta) = \beta + \beta(1-\alpha)^2 - 2(1-\alpha) < 0. \tag{6}$$

Figure 5 shows a surface plot of the positive part of the function $f(\alpha, \beta)$ for different α and β values. The negative portion of $f(\alpha, \beta)$ has been equated to zero in order to bring to light the range of α and β values for which the condition in equation (6) is violated. It is clear from the figure that causality would be violated for α and β values close to unity. One can, then, argue that local superluminal transmission of signals is impossible to achieve. This means that in

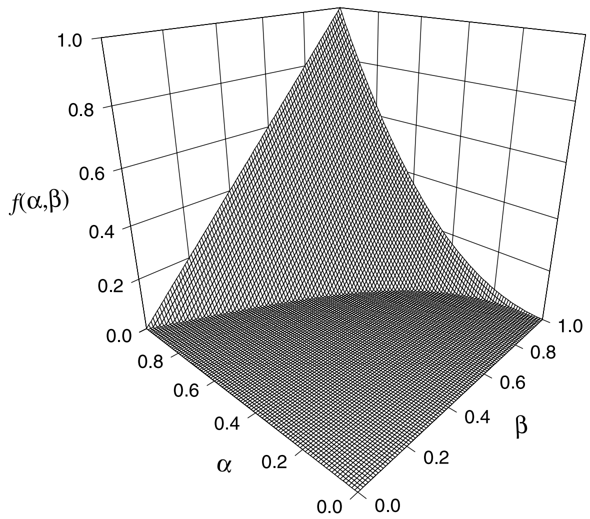

Figure 5. Surface plot showing ranges of α and β for which causality would be violated. Negative $f(\alpha, \beta)$ amplitudes, which correspond to α and β values that do not violate causality, are not plotted. Instead, such amplitudes are set to zero.

order to preserve causality, procedures similar to that illustrated in figure 2 should be ruled out. In particular, the shutter scheme is probably deemed to fail due to one of the reasons discussed in earlier paragraphs. Although ruling out local superluminal signalling is a viable option, one can argue that figure 5 demonstrates that communication with the past can only occur under extremely stringent conditions. It is clear from the figure that one could communicate with one's own past only when α or β approach unity. Either the primed and unprimed observers are moving at a relative velocity very close to c or that the 'delayed generation' takes a long time so that the following superluminal transmission is almost instantaneous. Both situations are very difficult to achieve physically. Thus, one may argue that local superluminal signalling is possible but is extremely difficult to use to communicate with the past.

4. Concluding remarks

Traditionally, superluminal signalling has been ruled out because, within the framework of special relativity, it would allow us to send information to the past. Since communication with the past has not been observed, then either special relativity is incorrect or superluminal signalling has to be dismissed. This work is concerned with the propagation of pulses that are locally superluminal, while entailing no violation of special relativity in a global sense. Examples of such pulses are pulsed Bessel beams investigated in [12]. In this paper, we have attempted to address the question of whether local superluminal pulses could be used for signalling without violating relativistic causality. We have described a scheme for local superluminal signalling that uses a shutter placed between a source of a train of pulsed Bessel beams and a receiver. Before accepting the validity of such scheme, we stress that the proposed shutter scheme should be tested. This could be done either by carrying out a comprehensive theoretical study of the effect of the shutter on the behaviour of the transmitted pulses, or by examining the scheme experimentally. In particular, one has to make sure that the action of the shutter will not destroy the pulses or their superluminality. This has not been done in this work. Instead, we considered straightaway the question of causality. It has been shown that if the proposed shutter scheme were feasible, the resulting superluminal signalling would entail a possible violation of causality, i.e. the shutter scheme would allow signalling to the past. However, we have shown that this could happen only if we used a source having a long 'delayed generation' period followed by almost instantaneous signalling, or by transferring information between two observers moving at relative velocities close to that of light. These situations are almost impossible to achieve. Consequently, one may use this result to provide a practical explanation for the reasons why signalling to the past has not been achieved. This line of reasoning should be contrasted with the *ad hoc* ruling out of superluminal signalling usually adopted to save causality. After all, communication to the past might turn out to be highly improbable instead of being impossible.

There have been recent reports on superluminal transmission of pulses by traversing undersized sections of waveguides, penetrating thin air gaps under conditions of frustrated total internal reflection and photonic-tunnelling through dielectric mirrors [21–31]. In these situations, the traversal time associated with the transmission of the peak of the pulse through the barrier region saturates at a constant value as the thickness of the barrier increases. This causes the speed of the pulse tunnelling through the barrier to appear to be superluminal. Akin to the case of pulsed Bessel beams, the superluminal propagation of tunnelling pulses takes place over finite distances. We are, thus, faced by another example of local superluminal transmission. It should be of interest to examine whether such tunnelling waves obey special relativity in a global sense. Following a similar analysis to the one adopted in this paper, we should be able to consider the effect of the use of tunnelling signals on relativistic causality. Furthermore, we can establish the type of constraints that would make communication to the past highly improbable. Already practical limitations to superluminal signalling using evanescent or tunnelling wavefields have been debated in the literature. Notable among these limitations are the effects of the finiteness of the bandwidths of the tunnelling pulses and the effects of simultaneously transmitting evanescent and non-evanescent field components [27, 29].

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References

[1] Lu J Y and Greenleaf J F 1992 Nondiffracting X waves—exact solutions to free space scalar wave equation and their finite aperture realization *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* **39** 19–31

[2] Lu J Y and Greenleaf J F 1992 Experimental verification of nondiffracting X waves *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* **39** 441–6

[3] Saari P and Sõnajalg H 1997 Pulsed Bessel beams *Laser Phys.* **7** 32–9

[4] Saari P and Reivelt K 1997 Evidence of X-shaped propagation-invariant localized light waves *Phys. Rev. Lett.* **79** 4135–8

[5] Ziolkowski R W, Besieris I M and Shaarawi A M 1993 Aperture realizations of the exact solutions to homogeneous-wave equations *J. Opt. Soc. Am.* A **10** 75–87

[6] Besieris I M, Abdel-Rahman M, Shaarawi A M and Chatzipetros A A 1998 Two fundamental representations of localized pulse solutions of the scalar wave equation *Prog. Electromagn. Res.* **19** 1–48

[7] Recami E 1998 On localized X-shaped superluminal solutions to Maxwell's equations *Physica* A **252** 586–610

[8] Mugnai D, Ranfagni A and Ruggeri R 2000 Observation of superluminal behaviors in wave propagation *Phys. Rev. Lett.* **84** 4830–3

[9] Rodriguez W A Jr and Lu J Y 1997 On the existence of undistorted progressive waves (UPWs) of arbitrary speeds $0 \leqslant v < \infty$ in nature *Found. Phys.* **27** 435

[10] Barashenkov V S and Rodriguez W A Jr 1998 Launching of non-dispersive sub- and superluminal beams *Nuovo Cimento* B **113** 329–38

[11] Capelas de Oliviera E and Rodriguez W A Jr 1998 Superluminal electromagnetic waves in free-space *Ann. Phys., Lpz.* **7** 654–9

[12] Shaarawi A M and Besieris I M 2000 On the superluminal propagation of X-shaped localized waves *J. Phys. A: Math. Gen.* **33** 7227–54 (preceding paper)

[13] Sõnajalg H, Rastep M and Saari P 1997 Demonstration of the Bessel-X pulse propagating with strong lateral and longitudinal localization in a dispersive medium *Opt. Lett.* **22** 310–2

[14] Bohm D 1965 *The Special Theory of Relativity* (Reading, MA: Benjamin-Cummings)

[15] Barut B O, Maccarrone G D and Recami E 1982 On the shape of tachyons *Nuovo Cimento* A **71** 509–33

[16] Recami E and Maccarrone G D 1980 Solving the 'imaginary quantities' problem in superluminal Lorentz transformations *Lett. Nuovo Cimento* **28** 151–7

[17] Caldirola P, Maccarrone G D and Recami E 1981 Second contribution on solving the 'imaginary quantities' problem in superluminal Lorentz transformations *Lett. Nuovo Cimento* **29** 241–50

[18] Recami E 1986 Classical tachyons and possible applications *Riv. Nuovo Cimento* **9** 1–178

[19] Recami E 1985 The Tolman–Regge 'Atitelephone' paradox: its solution by tachyon mechanics *Lett. Nuovo Cimento* **44** 587–93

[20] Recami E 1987 Tachyon mechanics and causality; a systematic thorough analysis of the tachyon causal paradoxes *Found. Phys.* **17** 239–96

[21] Nimtz G, Enders A and Spieker H 1994 Photonic tunneling times *J. Physique* **4** 1–10

[22] Enders A and Nimtz G 1992 On superluminal barrier traversal *J. Physique I* **2** 1693–8

[23] Enders A and Nimtz G 1993 Evanescent-mode propagation and quantum tunneling *Phys. Rev.* E **48** 632–5

[24] Enders A and Nimtz G 1993 Zero-time tunneling of evanescent mode packets *J. Physique* **3** 1089–92

[25] Nimtz G and Heitmann W 1997 Superluminal photonic tunneling and quantum electronics *Progr. Quant. Electron.* **21** 81–108

[26] Emig T 1996 Propagation of an electromagnetic pulse through waveguide with barrier: a time domain solution within classical electrodynamics *Phys. Rev.* E **54** 5780–7

[27] Nimtz G 1999 Evanescent modes are not necessarily Einstein causal *Eur. Phys. J.* B **7** 523–5

[28] Ph. Balcou and Dutraix L 1997 Dual optical tunneling times in frustrated total internal reflection *Phys. Rev. Lett.* **78** 851–4

[29] Carey J J, Zawadzka J, Jaroszynski D A and Klass Wynne 2000 Noncausal time response in frustrated total internal reflection *Phys. Rev. Lett.* **48** 1431–4

[30] Steinberg A M, Kwiat P G and Chiao R Y 1993 Measurement of the single-photon tunneling time *Phys. Rev. Lett.* **71** 708–11

[31] Laude V and Tournois P 1999 Superluminal asymptotic tunneling times through one-dimensional photonic bandgaps in quarter-wave-stack dielectric mirrors *J. Opt. Soc. Am.* B **16** 194–8